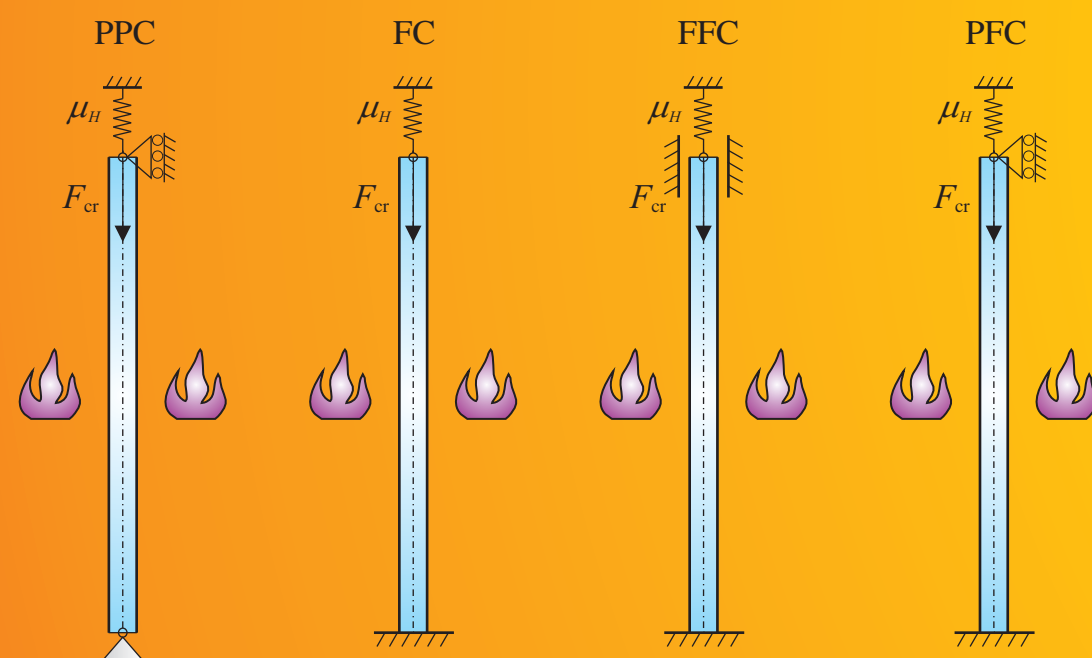




## Introduction

Steel columns are efficient structural elements both in terms of construction time and load bearing capacity. Steel is vulnerable to fire, however, and steel structures, potentially exposed to fire, require a particularly careful design. This especially holds true for steel columns as they are loaded in compression and are thus prone to buckling. With an increase of temperature, strength of steel and the stiffness of columns decrease leading to buckling at an even much lower level of external loading than at room temperature. We can find numerous results of experiments on steel columns in fire. Yet extensive parametric studies of behaviour of steel columns in fire can only be performed with numerical programs previously validated with the results of experiments. These programs are rather complex and not appropriate for a routine engineering usage. Therefore engineers will rather use simplified, practical methods such as those given in building codes, e.g. Eurocode 3, and BS595. These codes offer methods for the fire analysis for isolated columns, which, however, may not result in sufficiently reliable quantitative predictions of the fire bearing capacity of a column.

### Euler's columns exposed to fire



Presented analytical procedure for the determination of the critical temperature

### Preliminaries:

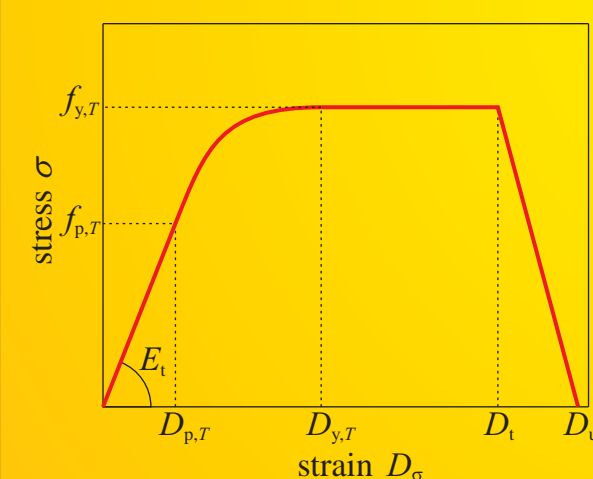
column is straight and geometrically perfect,  
temperature field is constant,

Kinematically exact planar beam model of Reissner:

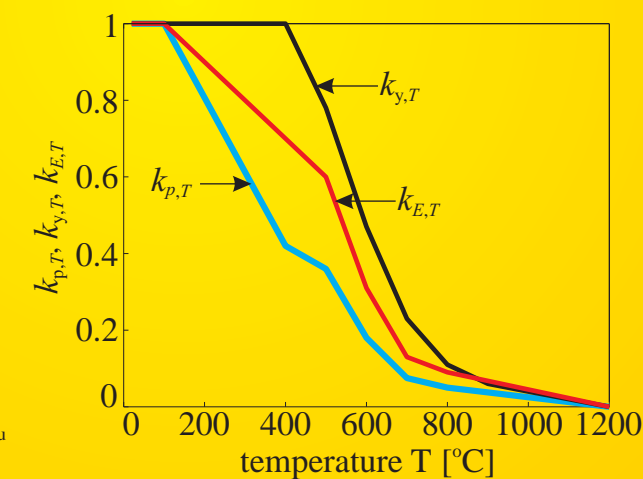
axial and flexural strains are considered  
shear strains are neglected

Nonlinear material model at elevated temperatures according to EC3 is used

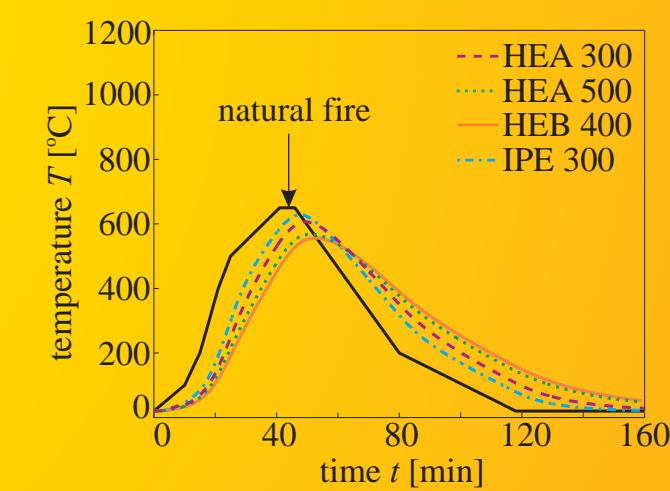
(a) constitutional model, EC3



(b) reduction factors, EC3



**Phase I**  
Determination of ambient temperature and temperature field over the cross-section of steel column;  
usage of fire curves, ISO834, Ec1,...;  
temperature field in column and its cross-section is constant  
simplified solution, Eurocode 3



### Procedure of fire analysis

**Phase II**  
Determination of the stress and strain field due to a combined effect of mechanical and temperature loads  
Principle of additivity of strains  
Reissner's geometrically exact beam theory  
corresponding governing equations of such a beam model are

$$\begin{aligned} 1+u'' - (1+\epsilon^i)\cos\varphi^i &= 0, \\ w'' + (1+\epsilon^i)\sin\varphi^i &= 0, \\ \varphi^{i'} - \kappa^i &= 0, \\ \mathcal{H}^{i'} &= 0, \\ \mathcal{V}^{i'} &= 0, \\ \mathcal{M}^{i'} - (1+\epsilon^i)Q^i &= 0, \\ \mathcal{N}^i = \mathcal{H}^i \cos\varphi^i - \mathcal{V}^i \sin\varphi^i &= 0, \\ Q^i = \mathcal{H}^i \sin\varphi^i + \mathcal{V}^i \cos\varphi^i &= 0, \\ \mathcal{N}^i = \int_A \sigma^i dA, \\ \mathcal{M}^i = \int_A z\sigma^i dA, \end{aligned}$$

## Determination of buckling load

linearization of the governing equations, around the fundamental solution ( $\varphi = 0$ )

After a systematic elimination of the unknowns is made, we end up with the system of two linear differential equations with constant coefficients:

$$\begin{aligned} \delta u^{i'''} &= 0, \\ \delta w^{j''''} + k^i \delta w^{j''} &= 0, \end{aligned}$$

with general solution

$$\begin{aligned} \delta u^i(x) &= \mathcal{K}_1^i x + \mathcal{K}_2^i, \\ \delta w^j(x) &= C_1^j \cos k^i x + C_2^j \sin k^i x + C_3^j x + C_4^j. \end{aligned}$$

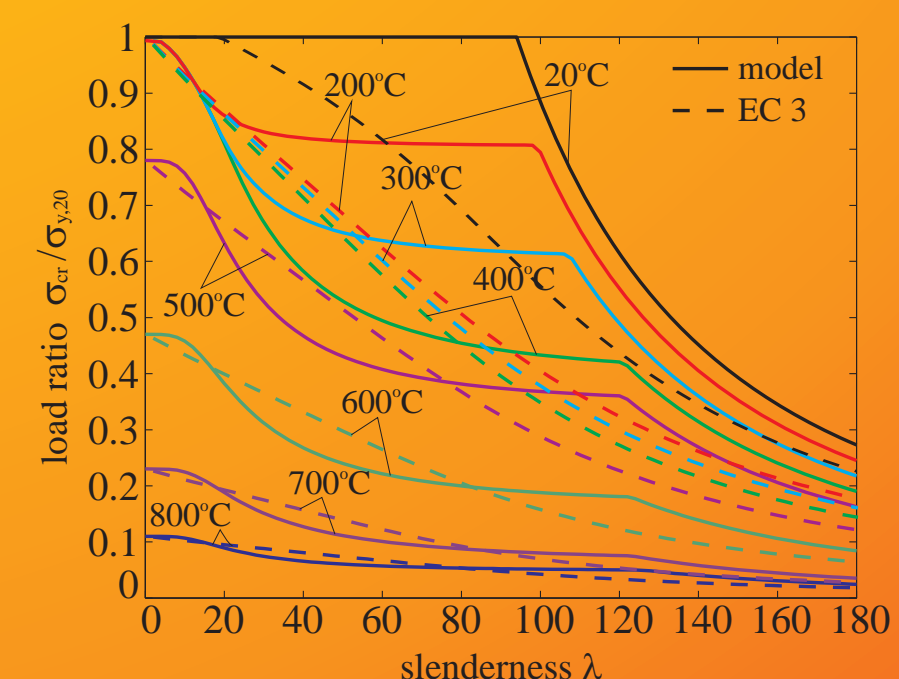
The non-trivial solution of the homogeneous system of linear algebraic equations is possible only if the determinant of the system matrix is zero

To determine the buckling load, the following set of non-linear algebraic equations for the three unknowns (critical axial force, critical axial strain and critical temperature) has to be solved:

$$\begin{aligned} \mathcal{N}_{cr} + F_{cr} + \mu_H \epsilon_{cr} L &= 0, \\ \mathcal{N}_{cr} - \sigma_{cr} A &= 0, \\ \det \mathbf{K}_\gamma &= 0. \end{aligned}$$

## Numerical example

Relationships between the critical stress ratio and the slenderness, at different temperatures



## Conclusions

Considering that steel at high temperature behaves in accordance with the material model proposed by EC 3, the critical temperature is determined exactly

simplified method proposed by EC 3 can be unsafe for moderate slendernesses

buckling appears to be the only mode of fracture of columns due to fire and

critical temperature highly depends on both the slenderness of a column and the material model of steel