Constitutive Equations for Structural Steel Subjected to Fire: Some Remarks

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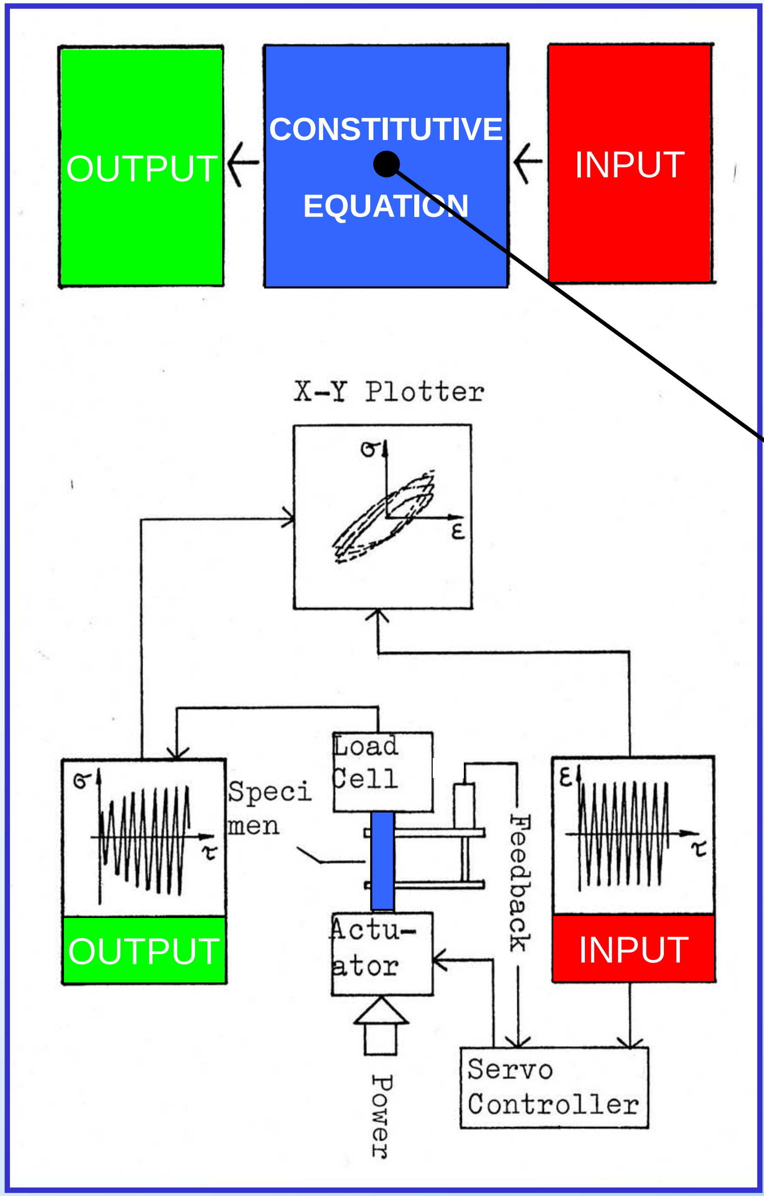
Operator view on material and constitutive equation



σ : Stress, ε : Strain, θ : Temperature

- F ***Operator***

$$\sigma(t) = \underset{0 \le \tau \le t}{\mathrm{F}} \left(\varepsilon(\tau), \theta(\tau) \right)$$

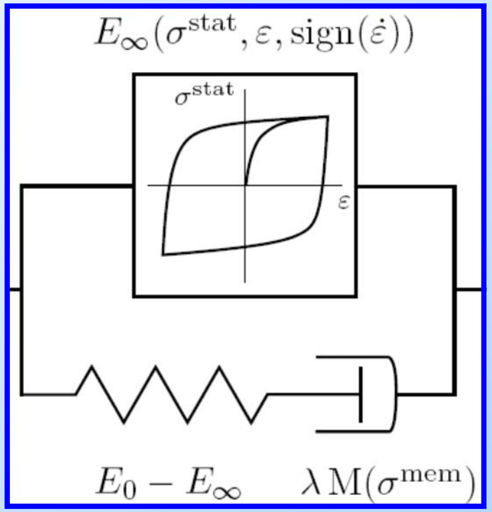
- $\varepsilon(\tau), \theta(\tau)$ ***Input* function (Loading)**
- $\sigma(t)$ ***Output* function (Response)**

Example of a ***constitutive model***

$$\dot{\sigma} = -\frac{1}{\lambda\left(\theta, \sigma - \sigma^{stat}\right)}\left[\sigma - \sigma^{stat}\right] + E_0\left(\theta\right)\dot{\varepsilon}^{mech} \quad (1)$$

$$\dot{\sigma}^{stat} = g\left(\theta, sign(\dot{\varepsilon}^{mech}), \varepsilon^{mech}, \sigma^{stat}\right)\dot{\varepsilon}^{mech} \quad (2)$$

$$\varepsilon^{mech} := \varepsilon - \alpha(\theta)\left[\theta - \theta_0\right] \quad (3)$$



- Derived from 3-parameter solid of viscoelasticity with modifications:

  - Strain replaced by mechanical strain
  - Linear elastic static stress replaced by rate-independent hysteresis
  - Dependence of all material parameters on temperature
  - Dependence of relaxation time on the difference between stress and static stress

- (1) – (3) ***unified*** constitutive model, no seperate creep and plastic strain
- (1) – (3) define an ***ODE*** system of type: $\underline{\dot{y}} = \underline{h}\left(\underline{y}(t), t\right), \underline{y}(0) = \underline{y}_0$

Transient creep (Basic capabilities)

- *Transient creep* is material response to special loading functions, i.e.

$$L_\sigma(\tau) = \sigma(\tau) = \sigma^* = const. \qquad (4)$$

and

$$L_\theta(\tau) = \theta(\tau) = \dot{\theta}^* \tau, \dot{\theta}^* = const. \qquad (5)$$

$$\dot{\varepsilon}^{mech} = \frac{\sigma^* - \sigma^{stat}}{\lambda\left(\theta, \sigma^* - \sigma^{stat}\right) E_0\left(\theta\right)} \qquad (6)$$

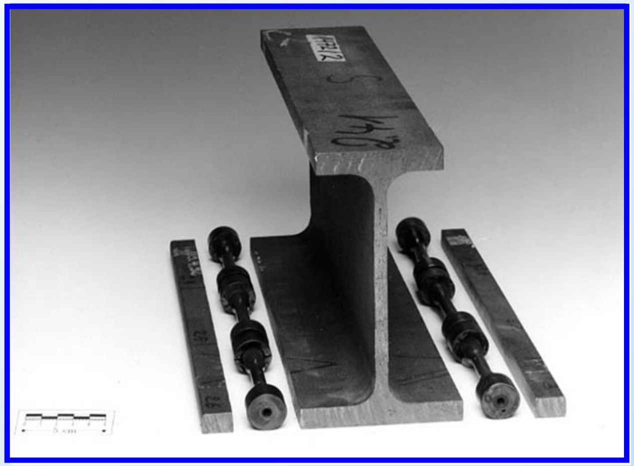
$$\dot{\sigma}^{stat} = g\left(\theta, sign(\dot{\varepsilon}^{mech}), \varepsilon^{mech}, \sigma^{stat}\right) \dot{\varepsilon}^{mech} \qquad (7)$$

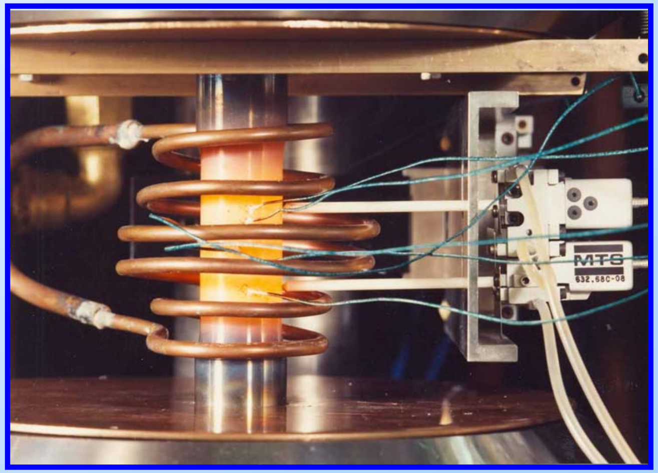
- Constitutive model as an operator to run numerical simulations via software package *RADAU* for different values of σ^* and $\dot{\theta}^*$

Results on poster: Figures 4, 5 and 6 !

- Special transient creep test:

  - Compression Loading: -100 MPa
  - LCF specimen produced from flange of an IPB 180 section, S235 steel



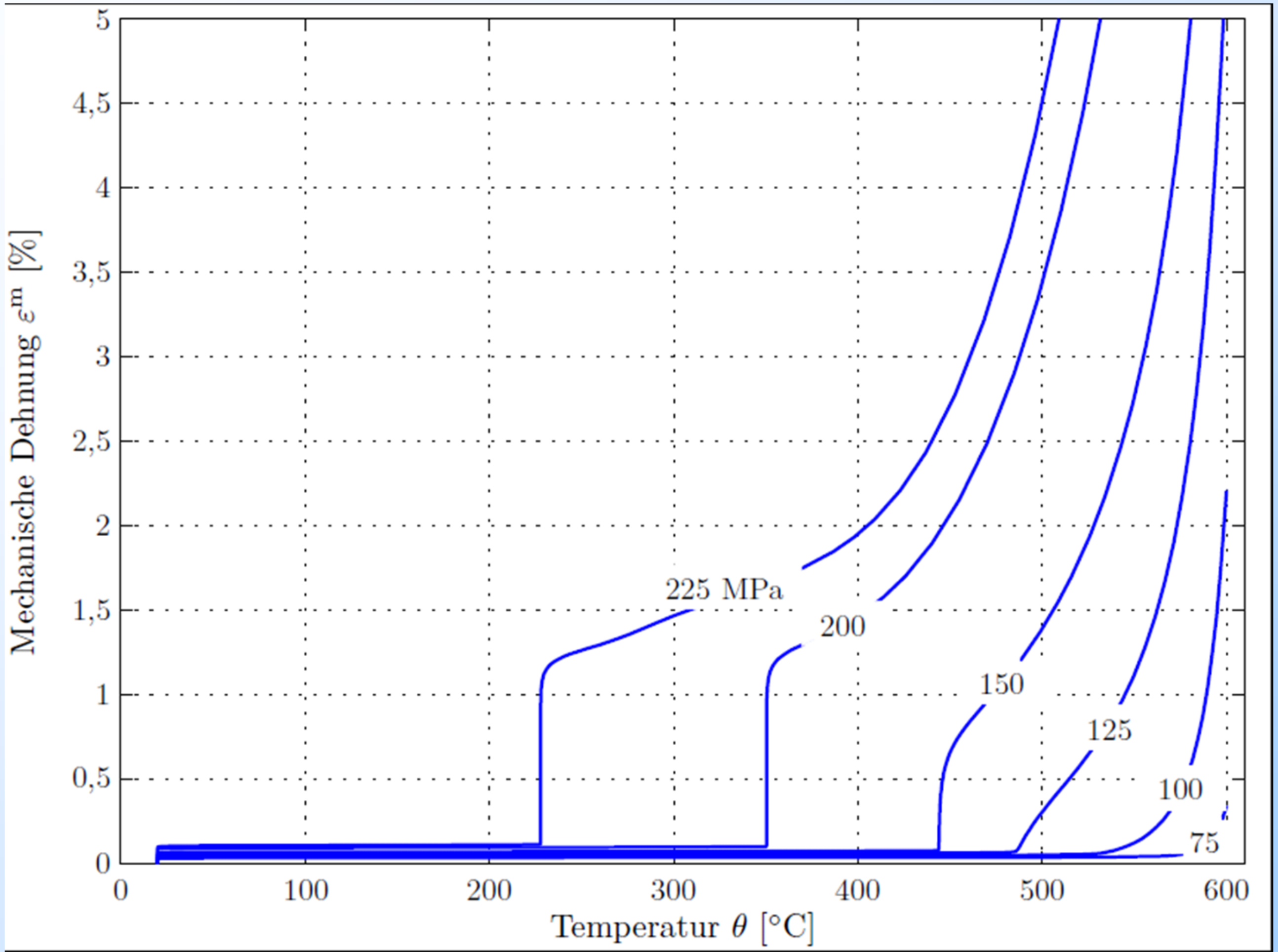


  - Non-monotonic heating through high-frequency inductive coils at 7 K/min

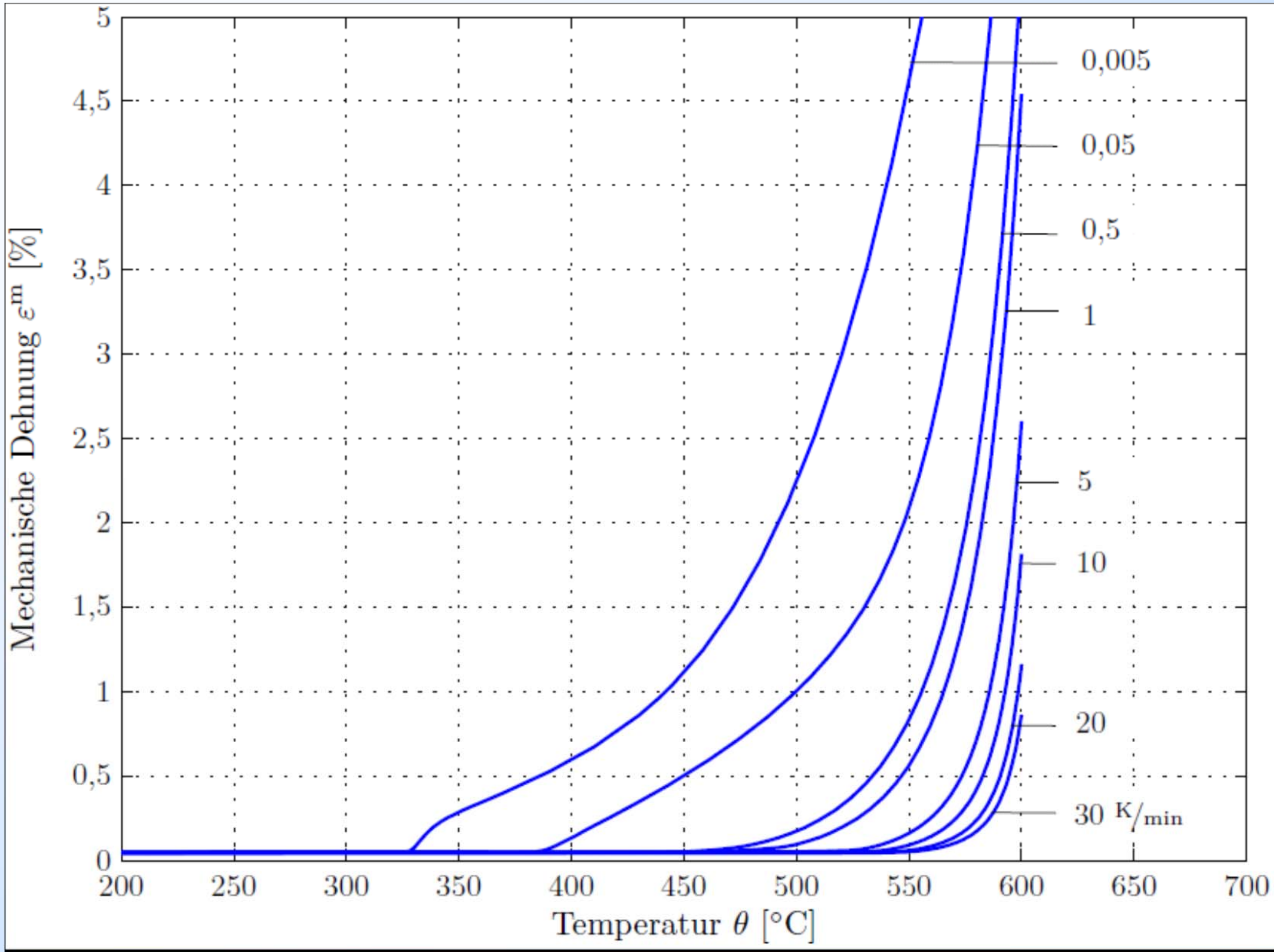
Results on poster: Figure 7 !

Thank you for your attention

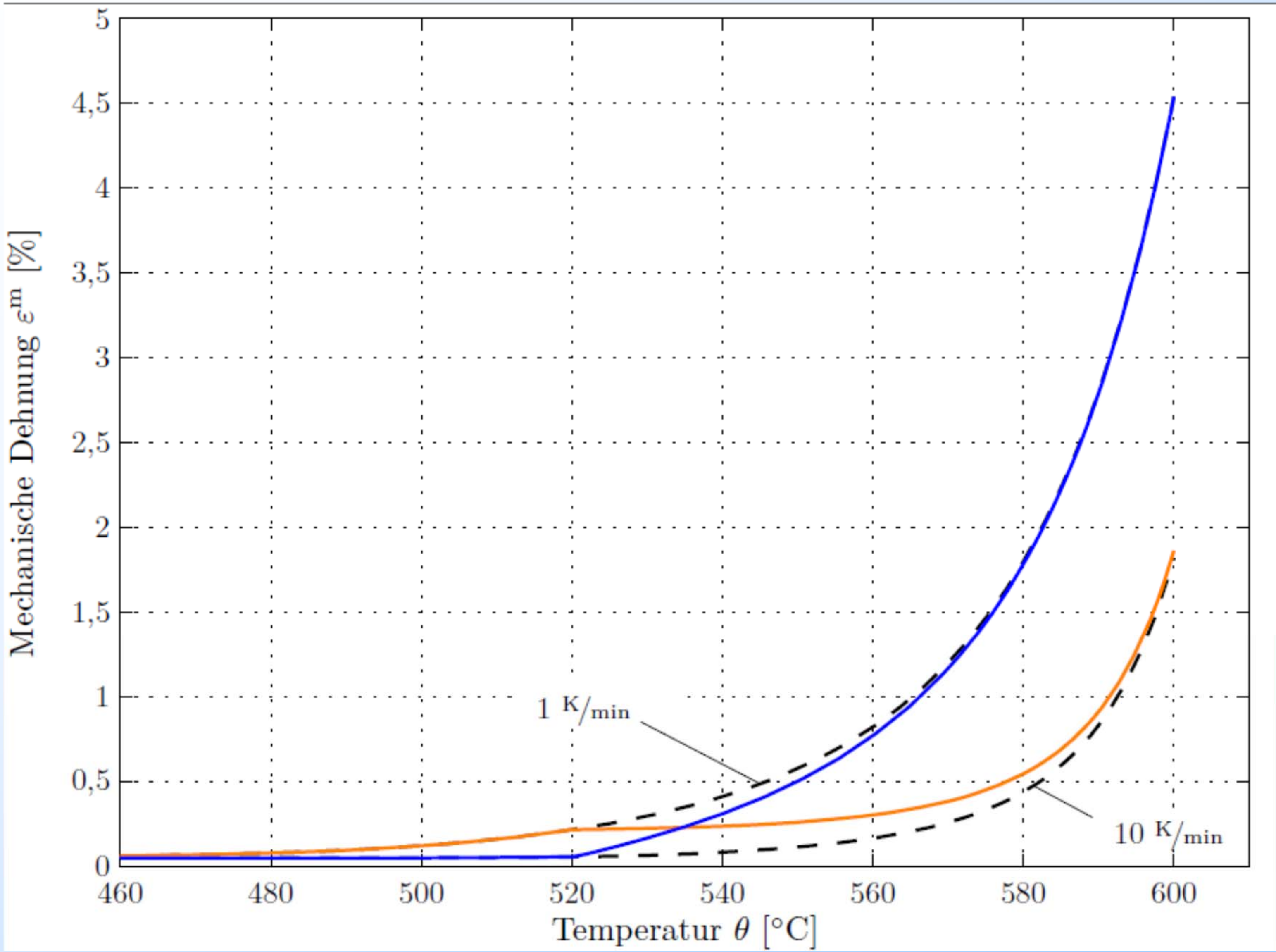
Acknowledgements: The author is very grateful to Dipl.-Ing. Le Trung Nguyen who has done all simulations including its visualisations.

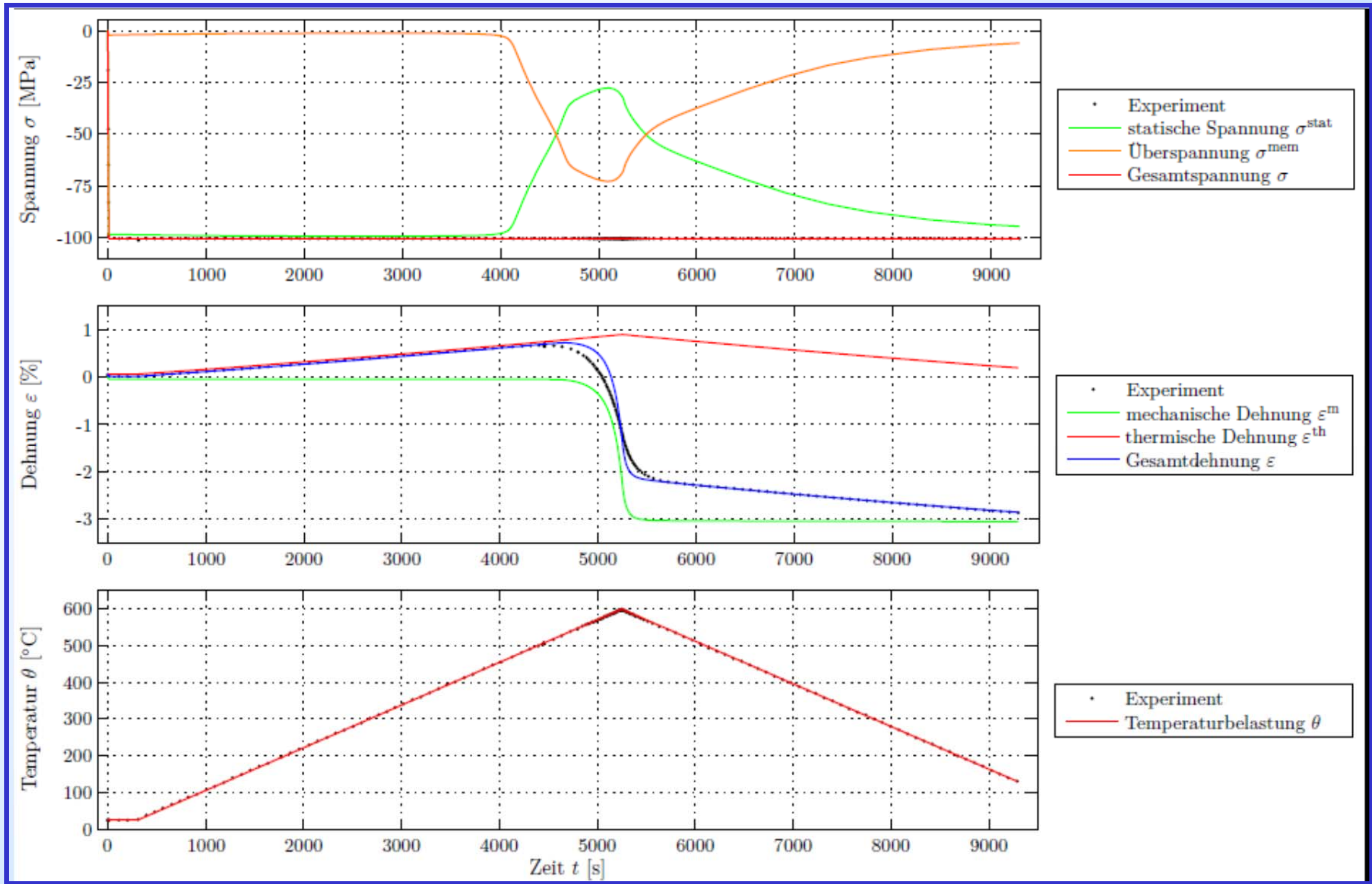


Mechanische Dehnung ε^m [%]
5
4,5
4
3,5
3
2,5
2
1,5
1
0,5
0
0
100
200
300
400
500
600
Temperatur θ [°C]
225 MPa
200
150
125
100
75



5
4,5
4
3,5
3
2,5
2
1,5
1
0,5
0
200
250
300
350
400
450
500
550
600
650
700
Mechanische Dehnung ε^{m} [%]
Temperatur θ [°C]
0,005
0,05
0,5
1
5
10
20
30 K/min


Mechanische Dehnung ε^m [%]
5
4,5
4
3,5
3
2,5
2
1,5
1
0,5
0
460
480
500
520
540
560
580
600
Temperatur θ [°C]
1 K/min
10 K/min



Spannung σ [MPa]
0
-25
-50
-75
-100
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
Experiment
statische Spannung σ^{stat}
Überspannung σ^{mem}
Gesamtspannung σ
Dehnung ε [%]
1
0
-1
-2
-3
Experiment
mechanische Dehnung ε^{m}
thermische Dehnung ε^{th}
Gesamtdehnung ε
Temperatur θ [°C]
600
500
400
300
200
100
0
Experiment
Temperaturbelastung θ
Zeit t [s]

$$\dot{\sigma} = -\frac{1}{\lambda\left(\theta,\sigma-\sigma^{stat}\right)}\left[\sigma-\sigma^{stat}\right] + E_0\left(\theta\right)\dot{\varepsilon}^{mech}, \sigma\left(0\right) = 0$$

$$\dot{\sigma}^{stat} = g\left(\theta, sign(\dot{\varepsilon}^{mech}), \varepsilon^{m}, \sigma^{stat}\right)\dot{\varepsilon}^{mech}, \sigma^{stat}\left(0\right) = 0$$

$$g\left(.\right) := E_e(\theta)\frac{\beta(\theta)E_e(\theta) - sign(\dot{\varepsilon}^{mech})\left[\sigma^{stat} - E_p(\theta)\varepsilon^{mech}\right]}{\beta(\theta)E_e(\theta) - \kappa(\theta)sign(\dot{\varepsilon}^{mech})\left[\sigma^{stat} - E_p(\theta)\varepsilon^{mech}\right]}$$

$$\varepsilon^{mech} := \varepsilon - \alpha(\theta)\left[\theta - \theta_0\right]$$

$$\underline{\dot{y}} = \underline{h}\left(\underline{y}(t), t\right), \underline{y}(0) = \underline{y}_0$$

$$\dot{\varepsilon}^{mech} = \frac{\sigma^{*} - \sigma^{stat}}{\lambda\left(\theta,\sigma-\sigma^{stat}\right)E_0\left(\theta\right)}, \varepsilon^{mech}\left(0\right) = 0$$

$$\dot{\sigma}^{stat} = g\left(\theta, sign(\dot{\varepsilon}^{mech}), \varepsilon^{mech}, \sigma^{stat}\right)\dot{\varepsilon}^{mech}, \sigma^{stat}\left(0\right) = 0$$