PART 5-3: Composite slab

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1 TASK

A composite slab has to be dimensioned in the fire situation. It is part of a shopping centre and the span is 4.8 m. The slab will be dimensioned as a series of simply supported beams. The required standard fire resistance class for the slab is R 90.

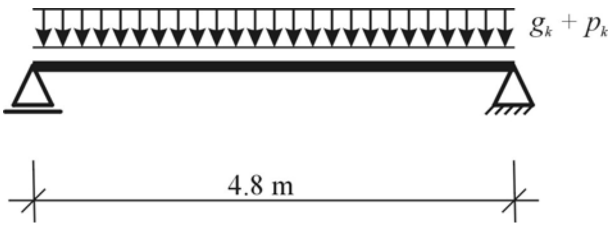

Figure 1. Static system

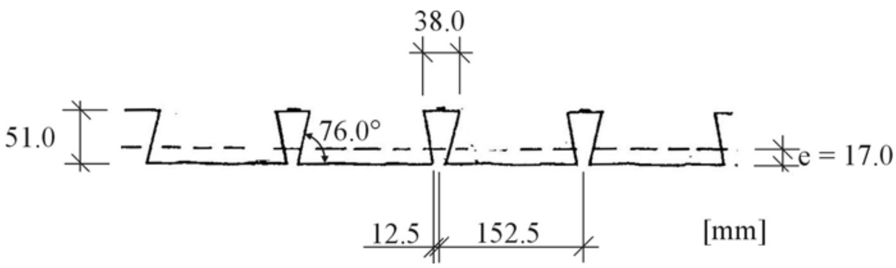

Figure 2. Steel sheet

Material properties
 Steel sheet:
  Yield stress: f_{yp} = 350 N/mm²
  Cross-sectional area: A_p = 1562 mm²/m
  Parameters for m+k method: k = 0.150 N/mm²
 Concrete:
  Strength category: C 25/30
  Compression strength: f_c = 25 N/mm²
  Height: h_t = 140 mm
  Cross-sectional area: A_c = 131,600 mm²/m

Loads:

Permanent loads:

Steel sheet	$g_{p,k}$	= 0.13 kN/m²
Concrete:	$g_{c,k}$	= 3.29 kN/m²
Finishing load:	$g_{f,k}$	= 1.2 kN/m²

Variable loads:

Live load:	p_k	= 5.0 kN/m²

Design sagging moment at ambient temperatures: $M_{s,d} = 39.56$ kNm

2 FIRE RESISTANCE OF A COMPOSITE SLAB

The composite slab has to be verified according to Section 4.3 and Annex D.

2.1 *Geometrical parameters and scope of application*

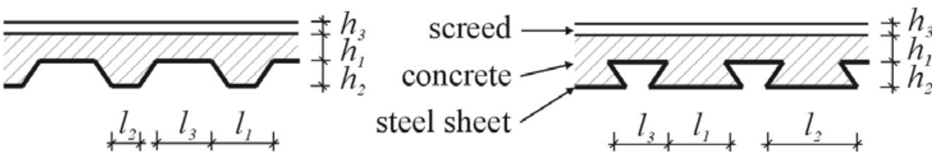

Figure 3. Geometry of cross-section

h_1 = 89 mm $\quad$ h_2 = 51 mm

l_1 = 115 mm $\quad$ l_2 = 140 mm $\quad$ l_3 = 38 mm

Table 1. Scope of application for slabs made of normal concrete and re-entrant steel sheets

Scope of application for re-entrant profiles [mm]	Existing geometrical parameters [mm]
$77.0 \le l_1 \le 135.0$	$l_1 = 115.0$
$110 \le l_2 \le 150.0$	$l_2 = 140.0$
$38.5 \le l_3 \le 97.5$	$l_3 = 38.0$
$50.0 \le h_1 \le 130.0$	$h_1 = 89.0$
$30.0 \le h_2 \le 70.0$	$h_2 = 51.0$

2.2 *Mechanical actions during fire exposure*

EN 1991-1-2

The load is determined by the combination rule for accidental situations.

$$E_{dA} = E\left(\sum G_k + A_d + \sum \psi_{2,i} \cdot Q_{k,i}\right)$$

Section 4.3

According to EN 1994 Part 1-2, the load E_d may be reduced by the reduction factor η_{fi}. It is calculated to:

EN 1994-1-2

$$\eta_{fi} = \frac{G_k + \psi_{2,1} \cdot Q_{k,1}}{\gamma_G \cdot G_k + \gamma_{Q,1} \cdot Q_{k,1}} = \frac{(0.13+3.29+1.2)+0.6\cdot 5.0}{1.35\cdot(0.13+3.29+1.2)+1.5\cdot 5.0} = 0.55$$

Section 2.4.2

With η_{fi}, the design bending moment $M_{fi,d}$ can be calculated:

$$M_{fi,d} = \eta_{fi} \cdot M_{sd} = 0.55 \cdot 39.56 = 21.76 \text{ kNm/m}$$

2.3 *Thermal insulation*

Section D.1

The thermal insulation criteria "I" has to ensure the limitation of the thermal condition of the member. The temperature on top of the slab should not exceed 140 °C in average and 180 °C at its maximum.

The verification is done in the time domain. The time in which the slab fulfils the criteria "I" is calculated to:

$$t_i = a_0 + a_1 \cdot h_1 + a_2 \cdot \Phi + a_3 \cdot \frac{A}{L_r} + a_4 \cdot \frac{1}{l_3} + a_5 \cdot \frac{A}{L_r} \cdot \frac{1}{l_3}$$

The rib geometry factor A/L_r is equivalent to the section factor A_p/V for beams. The factor considers that the mass and height have positive effects on the heating of the slab.

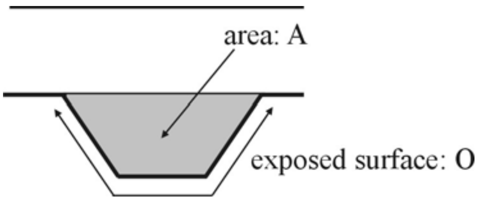

Figure 4. Definition of the rib geometry factor

$$\frac{A}{L_r} = \frac{h_2 \cdot \left(\frac{l_1 + l_2}{2}\right)}{l_2 + 2 \cdot \sqrt{{h_2}^2 + \left(\frac{l_1 - l_2}{2}\right)^2}} = \frac{51 \cdot \left(\frac{115 + 140}{2}\right)}{140 + 2 \cdot \sqrt{51^2 + \left(\frac{115 - 140}{2}\right)^2}} = 26.5\text{mm}$$

The view factor Φ considers the shadow effect of the rib on the upper flange.

$$\Phi = \left[\sqrt{{h_2}^2 + \left(l_3 + \frac{l_1 - l_2}{2}\right)^2} - \sqrt{{h_2}^2 + \left(+\frac{l_1 - l_2}{2}\right)^2}\right] \Big/ l_3$$

$$= \left[\sqrt{51^2 + \left(38 + \frac{115 - 140}{2}\right)^2} - \sqrt{51^2 + \left(\frac{115 - 140}{2}\right)^2}\right] \Big/ 38$$

$$= 0.119$$

The coefficients a_i for normal weight concrete is given in Table 2:

Table 2. Coefficients for determination of the fire resistance with respect to thermal insulation (see EN 1994-1-2, Annex D, Table D.1)

	a_0 [min]	a_1 [min/mm]	a_2 [min]	a_3 [min/mm]	a_4 mm·min	a_5 [min]
Normal weight concrete	-28.8	1.55	-12.6	0.33	-735	48.0
Light weight concrete	-79.2	2.18	-2.44	0.56	-542	52.3

With these parameters, t_i is calculated to:

$$t_i = (-28.8) + 1.55 \cdot 89 + (-12.6) \cdot 0.119$$
$$+ 0.33 \cdot 27 + (-735) \cdot 1/38 + 48 \cdot 27 \cdot 1/38$$
$$= 131.48 \text{ min} > 90 \text{ min} \quad \checkmark$$

2.4 *Verification of the load carrying-capacity*

Section 4.3.2

The plastic moment design resistance is calculated to:

$$M_{fi,t,Rd} = \sum A_i \cdot z_i \cdot k_{y,\theta,i} \cdot \left(\frac{f_{y,i}}{\gamma_{M,fi}} \right) + \alpha_{slab} \cdot \sum A_j \cdot z_j \cdot k_{c,\theta,j} \cdot \left(\frac{f_{c,j}}{\gamma_{M,fi,c}} \right)$$

To get the reduction factors $k_{y,\theta}$ for the upper flange, lower flange and the web, the temperatures have to be determined. These are calculated to:

Section D.2

$$\theta_a = b_0 + b_1 \cdot \frac{1}{l_3} + b_2 \cdot \frac{A}{L_r} + b_3 \cdot \Phi + b_4 \cdot \Phi^2$$

The coefficients b_i can be obtained from Table 3:

Table 3. Coefficients for the determination of the temperatures of the parts of the steel decking (see EN 1994-1-2, Annex D, Table D.2)

Concrete	Fire resis-tance [min]	Part of steel sheet	b_0 [°C]	b_1 [°C·mm]	b_2 [°C/mm]	b_3 [°C]	b_4 [°C]
Normal weight concrete	60	Lower flange	951	-1197	-2.32	86.4	-150.7
		Web	661	-833	-2.96	537.7	-351.9
		Upper flange	340	-3269	-2.62	1148.4	-679.8
	90	Lower flange	1018	-839	-1.55	65.1	-108.1
		Web	816	-959	-2.21	464.9	-340.2
		Upper flange	618	-2786	-1.79	767.9	-472.0
	120	Lower flange	1063	-679	-1.13	46.7	-82.8
		Web	925	-949	-1.82	344.2	-267.4
		Upper flange	770	-2460	-1.67	592.6	-379.0

For the different parts of the steel sheet, the temperatures are:
Lower flange:

$$\theta_{a,l} = 1018 - 839 \cdot \frac{1}{38} - 1.55 \cdot 27 + 65.1 \cdot 0.119 - 108.1 \cdot 0.119^2$$
$$= 960.29\ °C$$

Web:

$$\theta_{a,w} = 816 - 959 \cdot \frac{1}{38} - 2.21 \cdot 27 + 464.9 \cdot 0.119 - 340.2 \cdot 0.119^2$$
$$= 781.60\ °C$$

Upper flange:

$$\theta_{a,l} = 618 - 2786 \cdot \frac{1}{38} - 1.79 \cdot 27 + 767.9 \cdot 0.119 - 472.0 \cdot 0.119^2$$
$$= 580.87\ °C$$

To get the required load carrying-capacity during fire exposure, reinforcing bars have to be installed which normally are neglected for the ambient temperature design. For each rib, one reinforcing bar Ø 10 mm is chosen. The position of the bar can be seen in Figure 5.

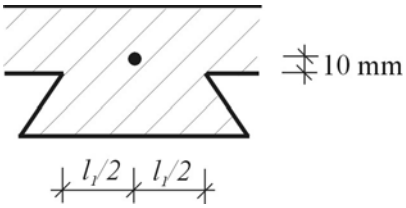

Figure 5. Arrangement of the reinforcing bar

The temperature of the reinforcing bar is calculated to:

$$\theta_s = c_0 + c_1 \cdot \frac{u_3}{h_2} + c_2 \cdot z + c_3 \cdot \frac{A}{L_r} + c_4 \cdot \alpha + c_5 \cdot \frac{1}{l_3}$$

where:

$$\frac{1}{z} = \frac{1}{\sqrt{u_1}} + \frac{1}{\sqrt{u_2}} + \frac{1}{\sqrt{u_3}}$$

$$= \frac{1}{\sqrt{l_1/2}} + \frac{1}{\sqrt{l_1/2}} + \frac{1}{\sqrt{h_2+10}} \quad \text{(simplified)}$$

$$= \frac{1}{\sqrt{57}} + \frac{1}{\sqrt{57}} + \frac{1}{\sqrt{61}}$$

$$= 0{,}393 \ 1/\text{mm}^{0.5}$$

$\Rightarrow \quad z = 2.54 \text{ mm}^{0.5}$

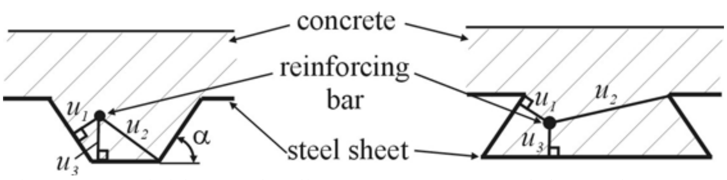

Figure 6. Definition of the distances u_1, u_2, u_3 and the angle α

The coefficients c_i for normal weight concrete is given in Table 4.

Table 4. Coefficients for the determination of the temperatures of the reinforcement bars in rib (see EN 1994-1-2, Annex D, Table D.3)

Concrete	Fire resistance [min]	c_0 [°C]	c_1 [°C]	c_2 [°C/mm$^{0.5}$]	c_3 [°C/mm]	c_4 [°C/°]	c_5 [°C]
Normal weight concrete	60	1191	-250	-240	-5.01	1.04	-925
	90	1342	-256	-235	-5.30	1.39	-1267
	120	1387	-238	-227	-4.79	1.68	-1326

With these parameters, the temperature of the reinforcing bar is:

$$\theta_s = 1342 + (-256) \cdot \frac{61}{51} + (-235) \cdot 2{,}54$$
$$+ (-5{,}30) \cdot 27 + 1{,}39 \cdot 104 + (-1267) \cdot \frac{1}{38}$$
$$= 407.0 \text{ °C}$$

For the steel sheet, the reduction factors $k_{y,i}$ are given in Table 3.2 of the EN 1994-1-2. For the reinforcement the reduction factor is given in Table 3.4, because the reinforcement bars are cold worked.

The carrying-capacity for each part of the steel sheet and the reinforcing bars can now be calculated.

Table 5. Reduction factors and carrying-capacities

	Temperature θ_i [°C]	Reduction factor $k_{y,i}$ [-]	Partial area A_i [cm²]	$f_{y,i}$ [kN/cm²]	Z_i [kN]
Lower flange	960.29	0.047	1.204	35.0	1.98
Web	781.60	0.132	0.904	35.0	4.18
Upper flange	580.87	0.529	0.327	35.0	6.05
Reinforcement	407.0	0.921	0.79	50.0	36.38

The plastic neutral axis is calculated as equilibrium of the horizontal forces. The equilibrium is set up for one rib ($b = l_1 + l_2$).

$$z_{pl} = \frac{\sum Z_i}{a_{slab} \cdot (l_1 + l_3) \cdot f_c} = \frac{1.98 + 4.18 + 6.05 + 36.38}{0{,}85 \cdot (115 + 38) \cdot 25 \cdot 10^{-3}} = 15.0 \text{ mm}$$

The plastic moment resistance for one rib is determined to:

Table 6. Calculation of the moment resistance of one rib

	Z_i [kN]	z_i [cm]	M_i [kNcm]
Lower flange	1.98	14.0	27.72
Web	4.18	14.0 – 5.1 / 2 = 11.45	47.86
Upper flange	6.05	14.0 – 5.1 = 8.9	53.85
Reinforcement	36.38	14.0 – 5.1 – 1,0 = 7.9	287.4
Concrete	-48.59	1.50 / 2 = 0.75	-36.44
			Σ 380.39

With the plastic moment of $M_{pl,rib}$ = 3.80 kNm and the width w_{rib} = 0.152 m of one rib, the plastic moment resistance of the composite slab is:

$$M_{fi,Rd} = 3.80/0.152 = 25.00 \text{ kNm/m}$$

Verification:

$$\frac{21.76}{25.00} = 0.88 < 1 \quad \checkmark$$

REFERENCES

EN 1991, *Eurocode 1:Actions on structures – Part 1-2: General actions – Actions on structures exposed to fire*, Brussels: CEN, November 2002

EN 1994, *Eurocode 4: Design of composite steel and concrete structures – Part 1-2: General Rules – Structural Fire Design*, Brussels: CEN, November 2006

QUALITY RECORD	WP5		
Title	Example to EN 1994 Part 1-2: Composite beam		
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