5. Imperfections

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Objectives of the lecture

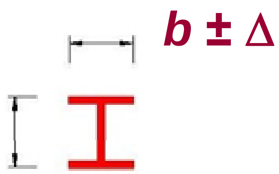
- This lecture describes various forms of imperfections of structures.
- It is shown, how these imperfections, whether at all, should be introduced into analysis.
- Finally some basic examples are presented.

Outline of the lecture

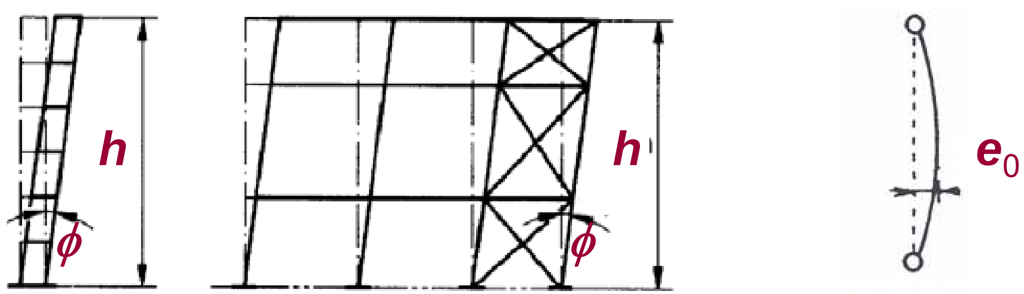
1. Types of imperfections
2. Introduction into analysis
3. Imperfections for global analysis
4. Imperfections of structures for analysis of bracings
5. Member imperfections
6. Imperfections versus tolerances
7. Examples
8. Conclusions

1. Types of imperfections

- Geometrical imperfections:
  - variance of dimensions of a structure or a member, e.g.:

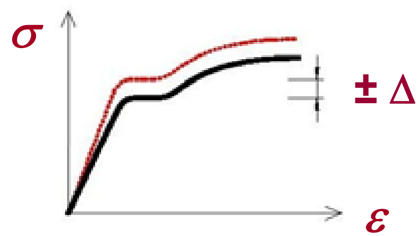


  - lack of verticality of a structure and straightness or flatness of a member, e.g.:

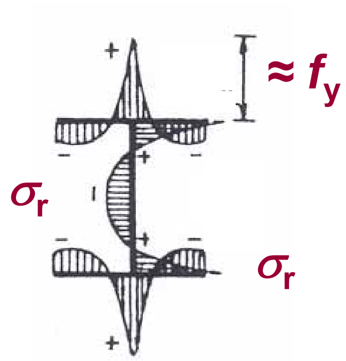


1. Types of imperfections

- Material imperfections:
  - variance of material properties, e.g.:



  - residual stresses (distribution in a cross section usually considered uniform along the member):

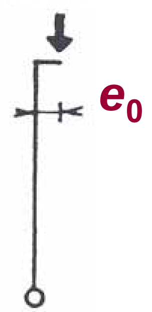


e. g. in I sections

(both hot-rolled and welded)

1. Types of imperfections

- Structural imperfections:
  - variance of boundary conditions, eccentricities in joints, e.g.:



2. Introduction into analysis

Introduction into analysis:

- In strut (frame) analysis all types of imperfections are usually introduced as **equivalent geometrical imperfections** (with increased value of amplitude e_{0d}).

- In plate (plated structure) analysis **geometrical imperfections and residual stresses** are introduced to derive buckling factors.

2. Introduction into analysis

In frame analysis the following imperfections shall be introduced:

- **Global imperfections** of frames or bracing systems
  (cover lack of verticality for frames or straightness of structure restrained by bracings)

- **Local (member) imperfections** of individual members
  (cover lack of straightness or flatness of a member and residual stresses of the member)

2. Introduction into analysis

- **Other imperfections** mentioned are covered by partial factors in the Limit State Design procedure.

In introduction of the equivalent geometrical imperfections (i.e. deflections) there is necessary to determine:

✓ shape of the deflection (buckling mode);

✓ amplitude of the deflection.

3. Imperfections for global analysis

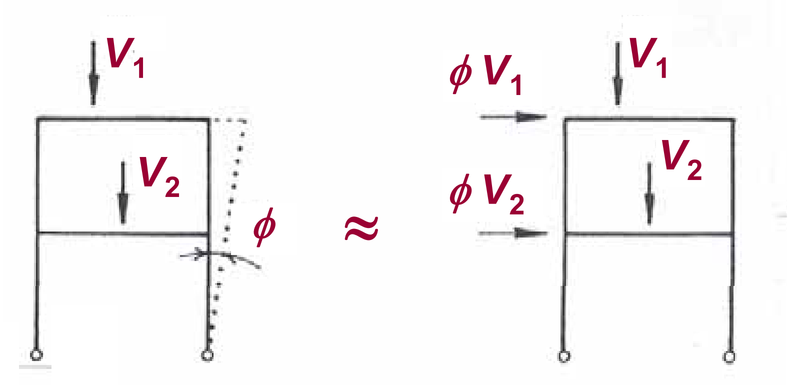
1. In general, the first critical buckling mode (η_{cr}) of the structure may be investigated and applied as imperfection shape for GNIA.

   The amplitude of the shape (e_{0d}) shall be determined from Eurocode 3, eq. 5.10, **securing required reliability** in the most axially stressed cross section.

2. Approximately, the global imperfection in sway mode (ϕ) and local geometrical imperfections (e_{0d}) of individual members may be introduced, see next page:

3. Imperfections for global analysis

- Global sway imperfections ϕ:

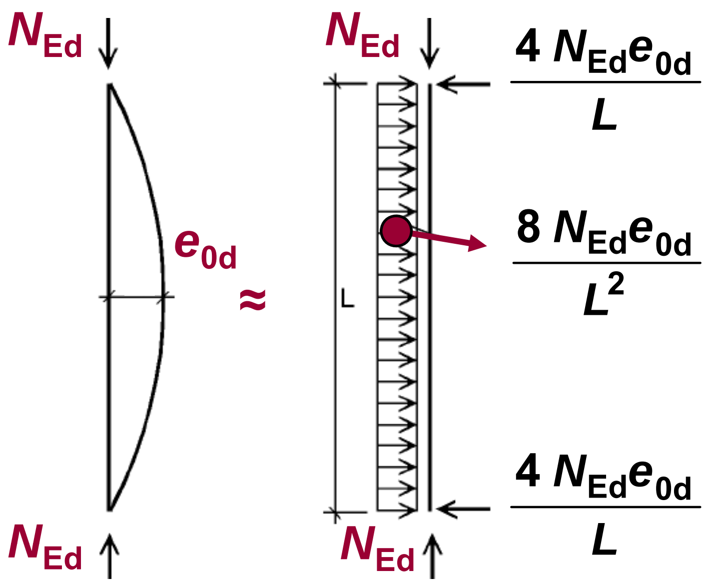


For value of ϕ see Eurocode 3, eq. 5.5. In general, the sway imperfections are introduced into analysis as corresponding horizontal loadings $H_i = \phi V_i$. Sway imperfections may be disregarded if the rate of horizontal/vertical loading is high (≥ 0,15), so that their contribution to internal forces is negligible.

3. Imperfections for global analysis

- Local geometrical imperfections

are given as values e_{0d}/L in Eurocode 3 (Tab. 5.1), which may be replaced by corresponding transverse uniform loadings giving the same bending moments.



3. Imperfections for global analysis

- Usually these local imperfections **are ignored in global analysis** and covered by reduction factors χ and χ_{LT} in member checks, unless the frame is sensitive to 2nd order effects, i.e.:

  - a member has at least one moment resistant end joint;

  - and has simultaneously high slenderness given in Eurocode 3, eq. 5.8.

4. Imperfections of structures for analysis of bracings

- Bracing systems may provide lateral stability of a strut in compression or a beam in bending. The strut/beam should be considered with a geometrical imperfection (initial bow) of amplitude $e_0 = L/500$ or less, taking number of strut/beams into account according to Eurocode 3, eq. 5.12.

- The bow with amplitude e_0 may be replaced by transverse uniform loading $q_{d.}$

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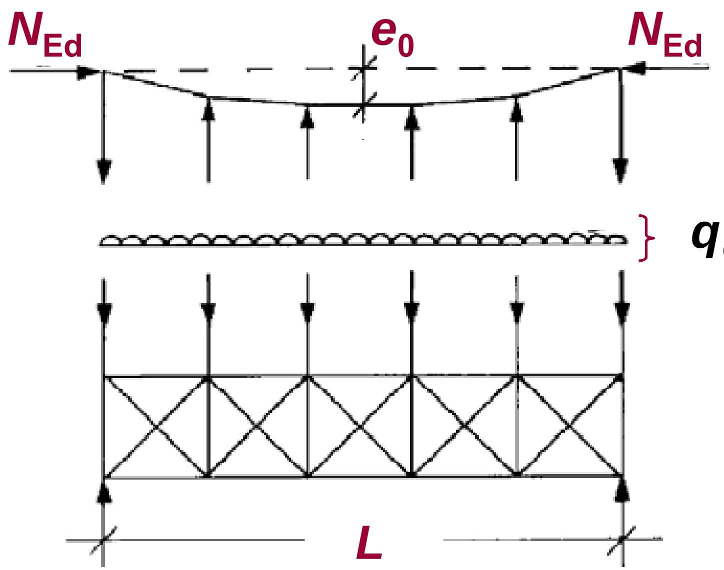
4. Imperfections of structures for analysis of bracings

- The loading q_d corresponds to impact of sum of the amplitude e_0 and the in-plane deflection of the bracing system δ_q. Such analysis requires 2nd order calculation or iterative procedure, see Eurocode 3, eq. 5.13:

member in compression (or compression flange force of a beam)

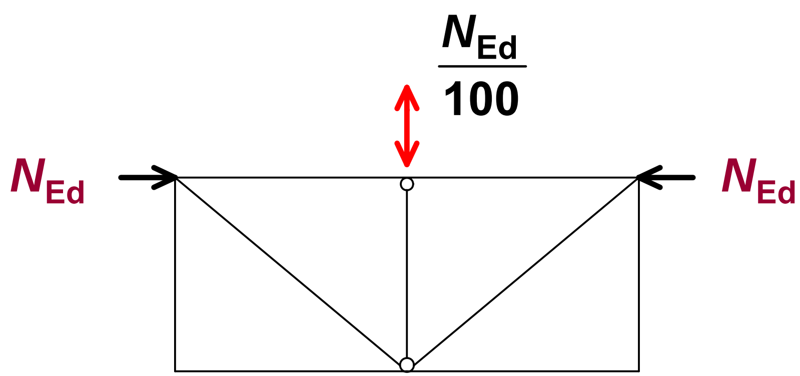
bracing system



$$q_d = N_{Ed} 8 \frac{e_0 + \delta_q}{L^2}$$

4. Imperfections of structures for analysis of bracings

- Members supporting a splice of compression members have to be verified for additional force $N_{Ed}/100$.



Formative Assessment Question 1

- Describe types of imperfections.
- How the imperfections are introduced into design of a steel structure?
- Describe form of global imperfections and their design model.
- Explain form of imperfection for bracings (e.g. rafter bracing in a roof of an industrial building) and how to encompass it for design.

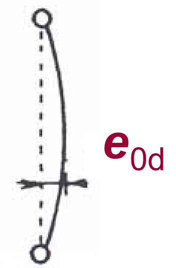
5. Member Imperfections

- In general, the influence of member (local) imperfections is covered by reduction factors (in columns and beams by χ, χ_{LT}, in plates by ρ, χ_w, χ_F).

- Instead, when using GNIA (geometrically nonlinear analysis with imperfections or approximate second order analysis), the imperfections of critical shape are taken with amplitudes as follows:

5. Member Imperfections

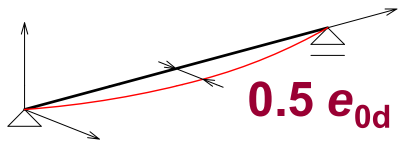
- For **compression struts** the equivalent initial bow may be used with form of flexural buckling and amplitude e_{0d} in accordance with Eurocode 3 (Tab. 5.1, e_{0d}/L given), e.g.:



L is system length;

e_{0d} depends on buckling curve and type of analysis.

- For **beams** in bending only equivalent initial bow in the direction of weak axis of the beam may be used, with amplitude 0.5 e_{0d} (where e_{0d} is as above):



5. Member Imperfections

- For **plates** the geometric imperfections should have amplitude of 80 % of assembly tolerances and residual stresses according to fabrication (say - compression stresses from 0.10 f_y to 0.25 f_y) or equivalent initial plate deflections with amplitude $b/200$ only and equivalent initial stiffener bow with amplitude $L/400$. For unstiffened compression plating e.g.:
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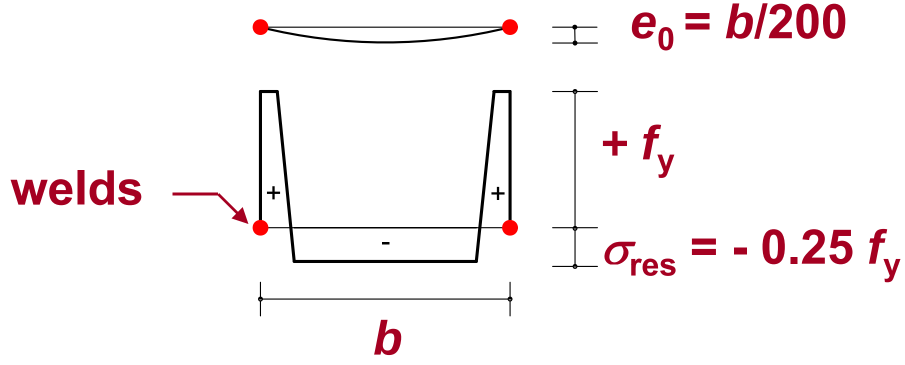
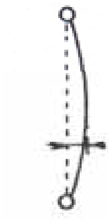


6. Imperfections versus tolerances

- Tolerances of structures/members are given in product standards and Eurocode EN 1090-2.
- Eurocode distinguishes **essential tolerances** (required for due resistance) and **functional tolerances** (class 1 and more rigorous class 2 for fit up and appearance requirements).
- **Essential tolerances** have to be confirmed by inspection and testing to determine quality of the structure.

6. Imperfections versus tolerances

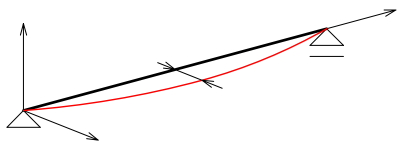
- Comparison of some imperfections e_{0d} for analysis and essential tolerances:
  - columns:



e_{0d} = *L*/100 ÷ *L*/350

tolerance: *L*/750

  - girders:

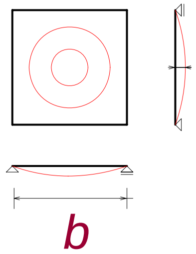


0,5 e_{0d}, i.e. *L*/200 ÷ *L*/700

tolerance: *L*/750

6. Imperfections versus tolerances

– unstiffened plates and stiffeners:



plates:

$e_{0d} = b/200$

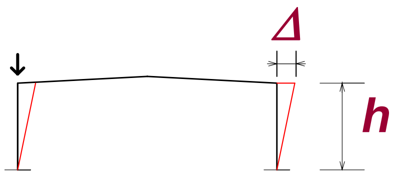
tolerance: $b/100$
(obviously incorrect)

stiffeners:

$e_{0d} = b/400$

tolerance: $b/400$

– frames, e.g. simple portal frame:



$\Delta_d = h/200$

tolerance: $h/500$

Formative Assessment Question 2

- How member imperfections are commonly introduced into design?
- How member imperfections are introduced into GNIA?
- Explain differences between imperfections and tolerances.

7. Examples

Example 1: Two-bay braced frame of a building

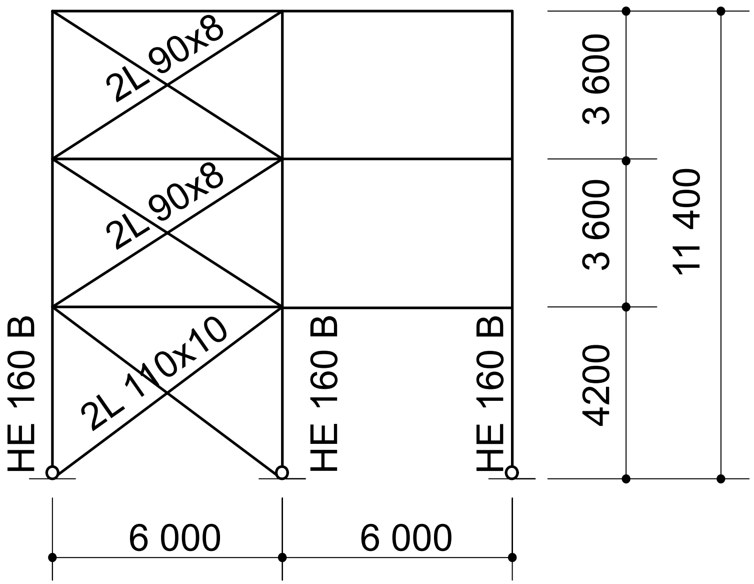
Example 2: Portal frame

Example 3: Rafter bracing

7.1 Example 1

Example 1:

Two-bay braced frame of a building



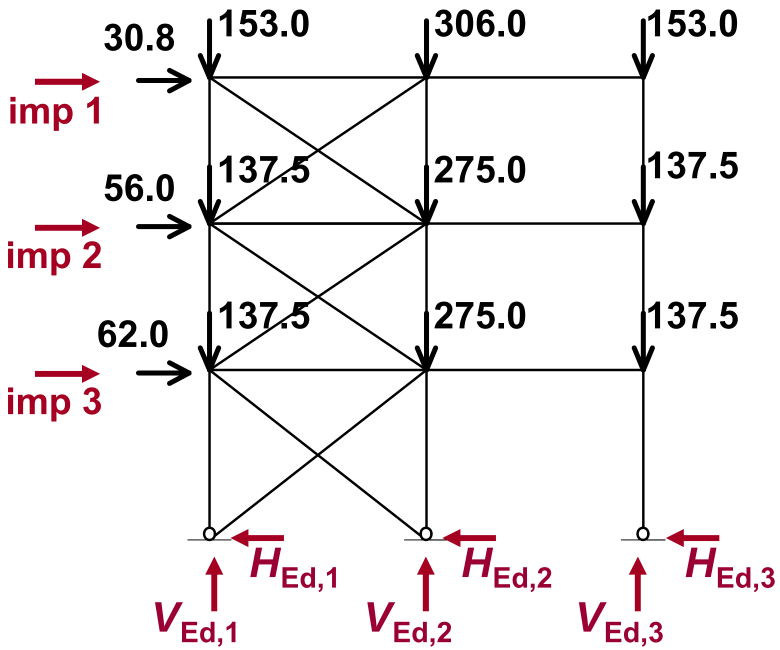
The frames spaced at distance of 6 m, bracing each 12 m.

Geometry and cross sections:

composite floor beams: A= 9345 mm^2, I = 127.4.10^6 mm^4

7.1 Example 1

Loading [kN] and reactions



Note: Wind (horizontal) loading is due to this bracing.

7.1 Example 1

- For formulas see Eurocode 3:
- ΣH_{Ed} = 148.8 kN i.e. < 0.15 ΣV_{Ed} = 256.8 kN (→ ϕ need to be considered)
- Sway imperfection for global analysis:

$$\alpha_h = \frac{2}{\sqrt{h}} = \frac{2}{\sqrt{11.4}} \quad \text{but} \quad \alpha_{h,min} = \frac{2}{3}$$

$$\alpha_m = \sqrt{0.5\left(1+\frac{1}{m}\right)} = \sqrt{0.5\left(1+\frac{1}{3}\right)} = 0.82$$

$$\phi = \phi_0\ \alpha_h\ \alpha_m = \frac{1}{200}\cdot\frac{2}{3}\cdot 0.82 = 0.0027$$

7.1 Example 1

- Global imperfections

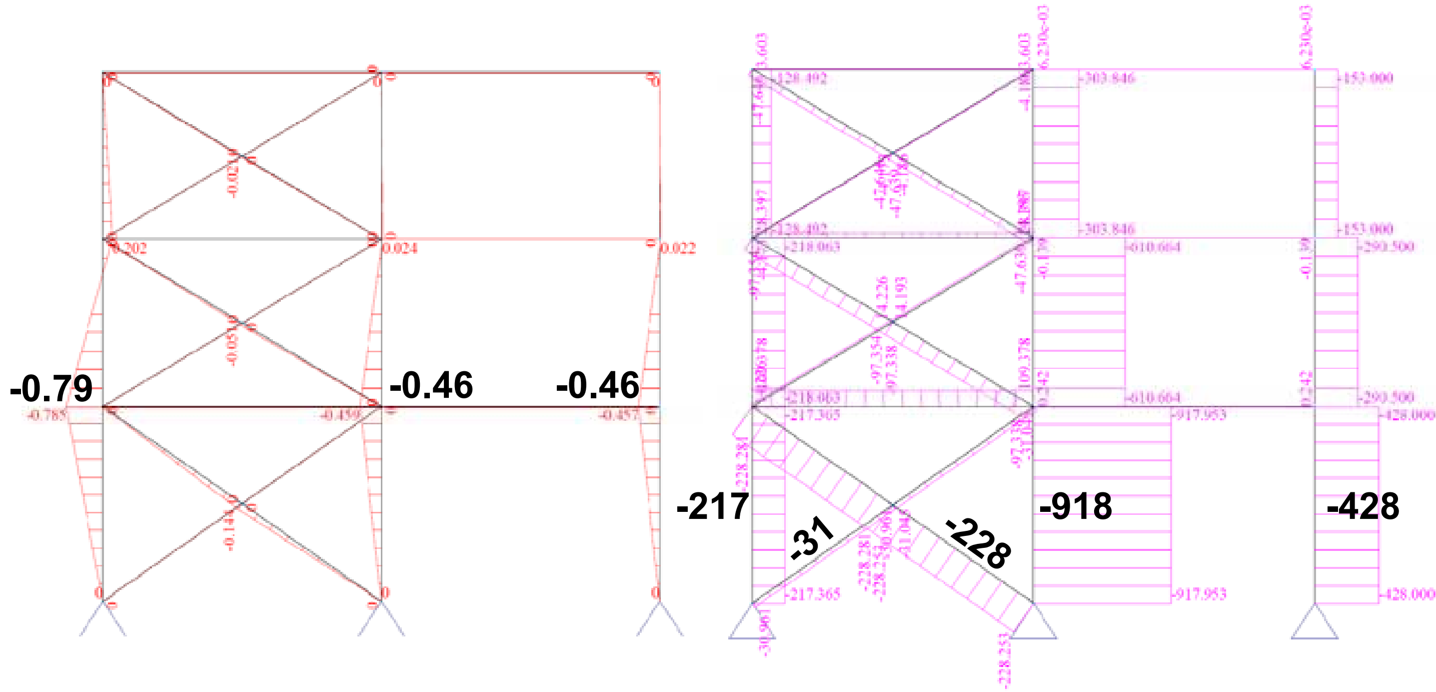
$$\text{imp } 1 = \phi \sum V = 0.0027 \cdot (153 + 306 + 153) = 1.6 \text{ kN}$$

$$\text{imp } 2 = \text{imp } 3 =$$
$$= \phi \sum V = 0.0027 \cdot (137.5 + 275 + 137.5) = 1.5 \text{ kN}$$

These imperfections belongs to each cross frame. For analysis of the bracing frame appropriate total values (as for wind loading) need to considered. Here they are doubled (belonging to two cross frames):

7.1 Example 1

- Internal forces due to loading + doubled imperfections:



M_{Ed} [kNm] N_{Ed} [kN]

7.1 Example 1

- Local imperfections for global analysis only if simultaneously (bottom central column):

  - exists moment resistant end joint: no ($M_{\mathrm{Ed}} \approx 0$)
  - slenderness

$$\bar{\lambda} > 0{,}5\sqrt{\frac{A f_{\mathrm{y}}}{N_{\mathrm{Ed}}}} = 0{,}5\sqrt{\frac{5425 \cdot 235}{918.0 \cdot 10^3}} = 0.60$$

true, because:

$$\bar{\lambda} = \frac{\lambda_{\mathrm{y}}}{\lambda_1} = \frac{4200 / 67.8}{93.9} = 0.66$$

- Therefore, the local imperfections could be ignored for global analysis in this example.

7.1 Example 1

- **Imperfections for local analysis**
- In LA (linear analysis) the local imperfections are covered by reduction factors (χ and χ_{LT}).

7.1 Example 1

- In GNIA analysis (see lecture 6 for details)
- The following imperfections should be used:

1. Generally either together with sway imperfections also approximate local (sinusoidal) bows with amplitudes in accordance with Eurocode 3, Tab. 5.1:

Buckling curve	Elastic analysis	Plastic analysis
	e_0 / L	
a_0	1/350	1/300
a	1/300	1/250
b	1/250	1/200
c	1/200	1/150
d	1/150	1/100

7.1 Example 1

- i.e. at columns (HE profile):

$$e_0 = L/250 = 4\,200/250 = 16.8 \text{ mm}$$

(buckling curve b)

- at composite beam approx.:

$$e_0 = L/200 = 4\,200/200 = 21.0 \text{ mm}$$

(buckling curve c)

- at bracing diagonals (L profile):

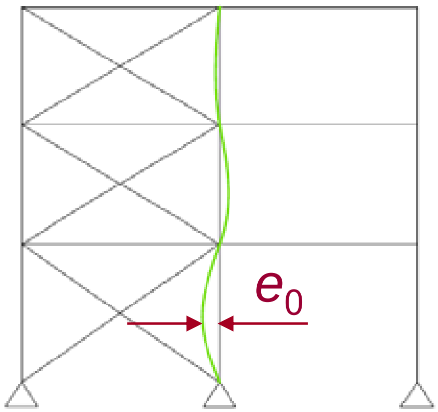
$$e_0 = L/250 = 3\,662/250 = 15.0 \text{ mm}$$

(buckling curve b)

Note: For this example however, the local imperfections can be ignored as shown above.

7.1 Example 1

2. Or unique global and local imperfection in the shape of the critical buckling mode (received from LBA) corresponding to buckling of **respective** member with amplitude e_0. The first buckling mode in the present frame corresponds to buckling of the central column (non-sway mode):



The first critical buckling mode:

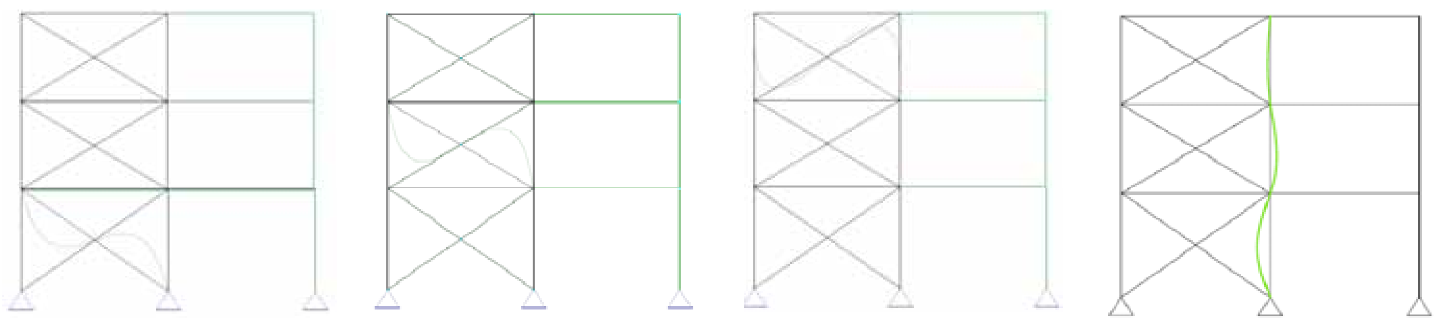
$$\alpha_{c,1} = 5.51$$

(Note: The first sway mode is the 15th, where $\alpha_{cr,15}$ = 144.08)

7.1 Example 1

Warning:
In some cases the first critical mode corresponds to other (less important) members, e.g. hinged diagonals. The critical mode corresponding to required member (e.g. column) may be of higher level. This higher mode shall be taken for column imperfections (**otherwise the solution is conservative**).

Example:
If in our frame the bottom diagonals are 2L 70x6 and overhead diagonals 2L 60x6, the 4th critical mode should be taken for column design:



1st mode $\alpha_{cr,1} = 1.66$ | 2nd mode $\alpha_{cr,2} = 2.16$ | 3rd mode $\alpha_{cr,3} = 4.80$ | 4th mode $\alpha_{cr,4} = 5.30$

7.1 Example 1

According to Eurocode 3, eq. 5.10:

$$e_0 = \alpha \left(\bar{\lambda} - 0.2\right) \frac{M_{\mathrm{Rk}}}{N_{\mathrm{Rk}}} \frac{1 - \dfrac{\chi \bar{\lambda}^2}{\gamma_{\mathrm{M1}}}}{1 - \chi \bar{\lambda}^2}$$

- where for central bottom column:

$$N_{\mathrm{c,Rk}} = A f_{\mathrm{y}} = 5425 \cdot 235 = 1275 \cdot 10^3 \text{ N}$$

$$\alpha_{\mathrm{ult,k}} = \frac{N_{\mathrm{c,Rk}}}{N_{\mathrm{Ed}}} = \frac{1275 \cdot 10^3}{918 \cdot 10^3} = 1.39$$

7.1 Example 1

- for diagonal 2 x L110/10 (other members not relevant):

$$N_{\mathrm{c,Rk}} = A f_{\mathrm{y}} = 4240 \cdot 235 = 996.4 \cdot 10^3 \text{ N}$$

$$\alpha_{\mathrm{ult,k}} = \frac{N_{\mathrm{c,Rk}}}{N_{\mathrm{Ed}}} = \frac{996.4 \cdot 10^3}{228.3 \cdot 10^3} = 4.4$$

Lower, i.e. column decides.

$$\bar{\lambda} = \sqrt{\frac{\alpha_{\mathrm{ult,k}}}{\alpha_{\mathrm{cr}}}} = \sqrt{\frac{1.39}{5.51}} = 0.50$$

For buckling curve b (α = 0.34): χ = 0.88

7.1 Example 1

$$M_{\mathrm{pl,Rk}} = W_{\mathrm{pl}} f_{\mathrm{y}} = 354 \cdot 10^3 \cdot 235 = 83.2 \cdot 10^6 \text{ kNm}$$

Resulting amplitude of imperfections in the shape of the first critical buckling mode:

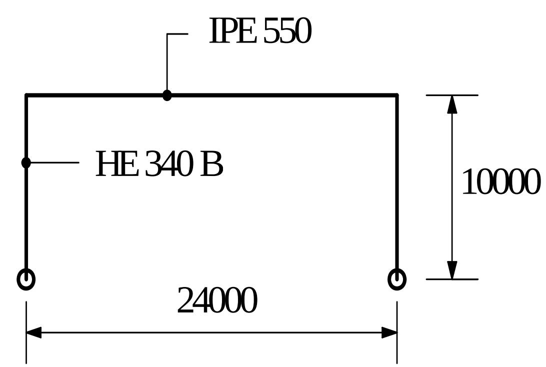
$$e_0 = \alpha \left(\bar{\lambda} - 0.2\right) \frac{M_{\mathrm{Rk}}}{N_{\mathrm{Rk}}} \frac{1 - \dfrac{\chi \bar{\lambda}^2}{\gamma_{M1}}}{1 - \chi \bar{\lambda}^2} =$$

$$= 0.34 \cdot (0.50 - 0.2) \cdot \frac{83.2 \cdot 10^6}{996.4 \cdot 10^3} \cdot \frac{1 - \dfrac{0.88 \cdot 0.50^2}{1.00}}{1 - 0.88 \cdot 0.50^2} = 8.5 \text{ mm}$$

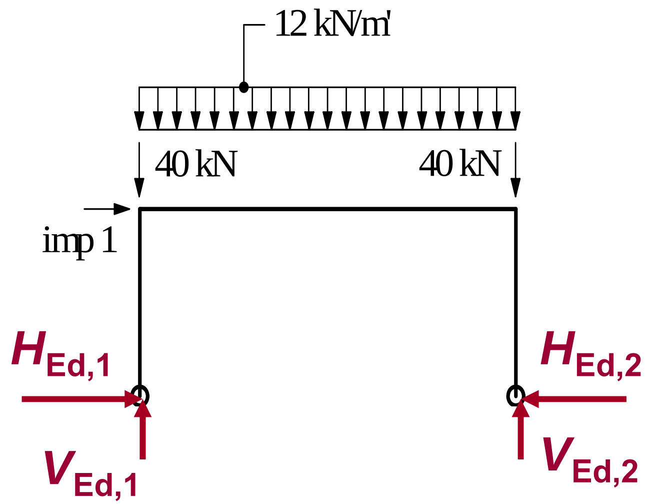
7.2 Example 2

Example 2: Portal frame

- **Imperfections for global linear analysis**



geometry and cross sections



loading and reactions

7.2 Example 2

- For formulas see Eurocode 3:
- $\Sigma\, H_{\text{Ed}} = 0$ i.e. $< 0.15\, V_{\text{Ed}}$ (consider ϕ)
- Sway imperfection for global analysis (imp 1):

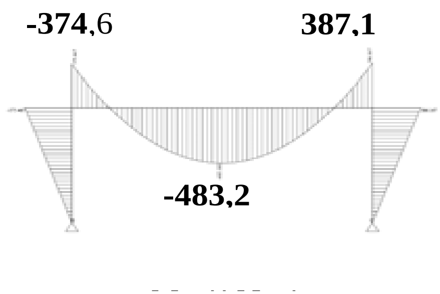
$$\alpha_h = \frac{2}{\sqrt{h}} = \frac{2}{\sqrt{10}} \quad \text{but} \quad \alpha_{h,min} = \frac{2}{3}$$

$$\alpha_{\text{m}} = \sqrt{0.5\left(1+\frac{1}{m}\right)} = \sqrt{0.5\left(1+\frac{1}{2}\right)} = 0.87$$

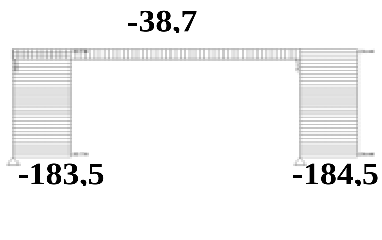
- $\phi = \phi_0\, \alpha_{\text{h}}\, \alpha_{\text{m}} = \frac{1}{200}\cdot\frac{2}{3}\cdot 0.87 = 0.0029$
- imp 1 $= \phi \sum V = 0.0029 \cdot (12 \cdot 24 + 80) = 1.07$ kN

7.2 Example 2

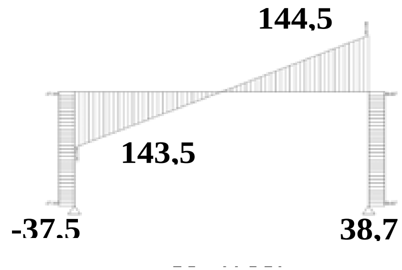
- Internal forces (loading + imperfections):



M_{Ed} [kNm]



N_{Ed} [kN]



V_{Ed} [kN]

7.2 Example 2

- Local imperfections for global analysis only if simultaneously (column concerned):
  - exists moment resistant end joint: OK
  - slenderness

$$\bar{\lambda} > 0{,}5\sqrt{\frac{A f_y}{N_{Ed}}} = 0{,}5\sqrt{\frac{17090 \cdot 235}{184.5 \cdot 10^3}} = 2{,}33$$

not true, because

$$\bar{\lambda} = \frac{\lambda_y}{\lambda_1} = \frac{10000 / 146.5}{93.9} = 0.73$$

- There, the local imperfections can be ignored in global analysis.

7.2 Example 2

- **Imperfections for local analysis**

- In LA (linear analysis) the local imperfections are covered by reduction factors (χ and χ_{LT}).

7.2 Example 2

- Alternative GNIA analysis
- In GNIA the following imperfections should be used:

1. Generally either together with sway imperfections also approximate local (sinusoidal) bows with amplitudes in accordance with Eurocode 3, Tab. 5.1:

at columns: e_0 = L/250 = 10000/250 = 40 mm

(buckling curve b)

at beam: e_0 = L/300 = 24000/300 = 80 mm

(buckling curve a)

Note: For this example however, the local imperfections can be ignored as shown above.

7.2 Example 2

2. Or unique global and local imperfection in the shape of the first critical buckling mode received from LBA, with amplitude e_0:

The first critical buckling mode:
$\alpha_{cr,1} = 6.93$

7.2 Example 2

According to Eurocode 3, eq. 5.10:

$$e_0 = \alpha \left(\bar{\lambda} - 0.2\right) \frac{M_{Rk}}{N_{Rk}} \frac{1 - \frac{\chi \bar{\lambda}^2}{\gamma_{M1}}}{1 - \chi \bar{\lambda}^2}$$

- where for columns:

$$N_{c,Rk} = A f_y = 17090 \cdot 235 = 4016 \cdot 10^3 \text{ N}$$

$$\alpha_{ult,k} = \frac{N_{c,Rk}}{N_{Ed}} = \frac{4016 \cdot 10^3}{184.5 \cdot 10^3} = 21{,}8$$

7.2 Example 2

- for beam:

$$N_{c,Rk} = A f_y = 13440 \cdot 235 = 3158 \cdot 10^3 \text{ N}$$

$$\alpha_{ult,k} = \frac{N_{c,Rk}}{N_{Ed}} = \frac{3158 \cdot 10^3}{384.7 \cdot 10^3} = 81{,}6$$

Lower, i.e. column decides.

$$\bar{\lambda} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr}}} = \sqrt{\frac{21{,}8}{6.93}} = 1.77$$

For buckling curve b (α = 0.34): χ = 0.26

7.2 Example 2

$$M_{\mathrm{pl,Rk}} = W_{\mathrm{pl}} f_{\mathrm{y}} = 2408 \cdot 10^{3} \cdot 235 = 565.9 \cdot 10^{6} \text{ kNm}$$

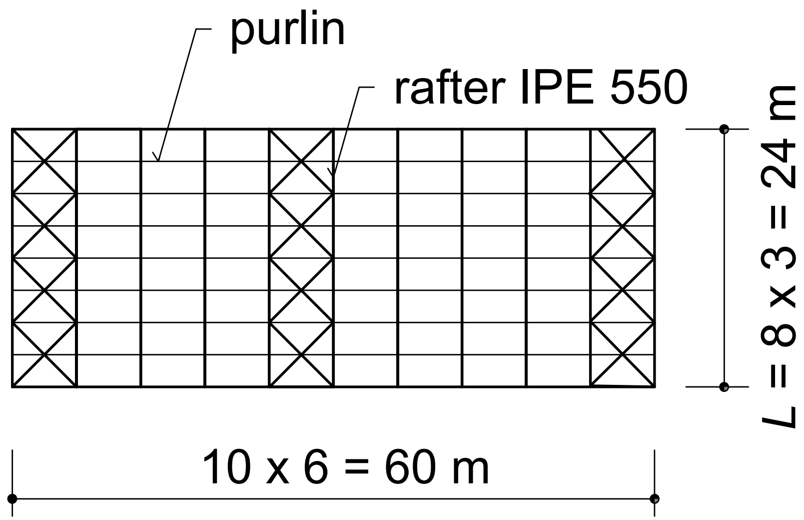
Resulting amplitude of imperfections in the shape of the first critical buckling mode:

$$e_0 = \alpha \left(\bar{\lambda} - 0.2\right) \frac{M_{\mathrm{Rk}}}{N_{\mathrm{Rk}}} \frac{1 - \dfrac{\chi \bar{\lambda}^2}{\gamma_{M1}}}{1 - \chi \bar{\lambda}^2} =$$

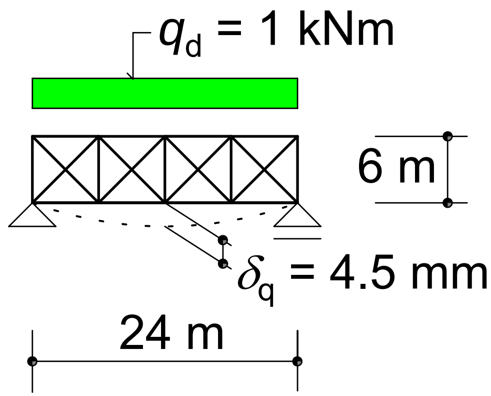
$$= 0.34 \cdot (1.77 - 0.2) \cdot \frac{565.9 \cdot 10^{6}}{4016 \cdot 10^{3}} \cdot \frac{1 - \dfrac{0.26 \cdot 1.77^2}{1.00}}{1 - 0.26 \cdot 1.77^2} = 75.2 \text{ mm}$$

7.3 Example 3

Example 3: Rafter bracing



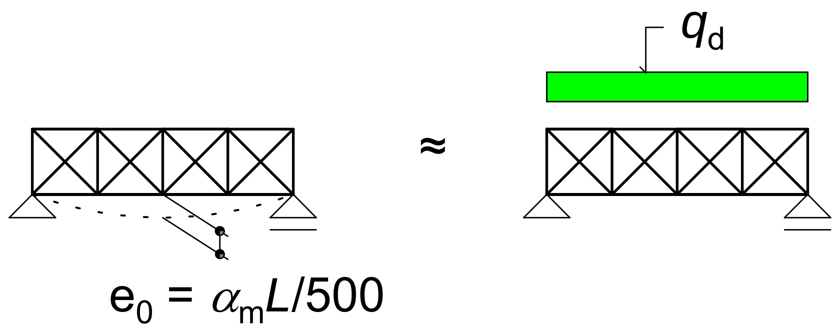
Plan of the roof



Deflection of bracing

7.3 Example 3

- Initial deflections with amplitude e_0 of the bracing system will be replaced by equivalent stabilizing force q_d:



Data from former calculations:

- max. moment in the rafter: M_{Ed} = 362.0 kNm
- max. force in the compression flange of the rafter:
  $N_{Ed} = M_{Ed}/h = 362/0.5328 = 679.4$ kN
- external loading per one bracing system: $q_{d,ext}$ = 3.70 kN/m

Number of braced flanges per one bracing system: m = 11/3 = 3.67

7.3 Example 3

- Amplitude e_0:

$$\alpha = \sqrt{0.5\left(1+\frac{1}{m}\right)} = \sqrt{0.5 \cdot \left(1+\frac{1}{3.67}\right)} = 0.80$$

$$e_0 = \alpha_{\mathrm{m}} L / 500 = 0.80 \cdot 24000 / 500 = 38.4 \text{ mm}$$

- Equivalent stabilizing loading q_{d} requires iterative procedure. To avoid the iteration, suitable guess of the total deflection $\delta_{\mathrm{q,(0)}}$ from stabilizing loading q_{d} and all external loading $q_{\mathrm{d,ext}}$ is necessary. Say:

$$\delta_{\mathrm{q(0)}} \approx \frac{L}{500} = 48.0 \text{ mm}$$

7.3 Example 3

and therefore the equivalent stabilizing loading :

$$q_d = \sum N_{\mathrm{Ed}} 8 \frac{e_0 + \delta_{\mathrm{q(0)}}}{L^2} = \left(3.67 \cdot 679.4 \cdot 10^3\right) \cdot 8 \cdot \frac{38.4 + 48.0}{24000^2} = 2.99\ \mathrm{N/mm}$$

Check of the guess of $\delta_{\mathrm{q(0)}}$:

$$\delta_{\mathrm{q}(1)} = \left(q_{\mathrm{d}} + q_{\mathrm{d,ext}}\right)\delta_{\mathrm{q(q=1)}} = \left(3.70 + 2.99\right) \cdot 4.5 = 30.1\,\mathrm{mm}$$

The guess was OK, because conservative:

$\delta_{\mathrm{q(0)}}$ = 48.0 mm > $\delta_{\mathrm{q(1)}}$ = 30.1 mm

8. Conclusions

1) Imperfections significantly influence strength of structures.
2) In frame structures equivalent geometrical imperfections (initial deflections) may substitute all kinds of imperfections.
3) In plated structures preferably both initial deflections and residual stresses should be introduced into design.
4) Generally, global and local imperfections have to be considered.

8. Conclusions

5) Shape of the initial deflections is generally given by the first critical mode, approximately in the form of initial sway imperfection and individual bow imperfections of members.

6) Amplitude of the initial imperfections shall correspond to values given in Eurocode 3 (chapter 5.3.2) to secure required reliability of design.

7) In common design, the influence of imperfections is usually covered by global geometrical imperfections and reduction factors for members.

8) Compression residual stresses shall correspond to expected mean values.

Notes to Users of the Lecture

- This session is for imperfections of structures and requires about 60 minutes lecturing and 60 minutes for tutorial session.
- Within the lecturing, three types of imperfections necessary to account for in analysis of a structure are described. In particular, introduction of global imperfections, imperfections for bracing systems and imperfections of individual members in compression and bending are shown. Attention is also paid to tolerances required by Eurocode EN 1090 for execution.
- Further readings on the relevant documents from website of www.access-steel.com and relevant standards of national standard institutions are strongly recommended.
- Formative questions should be well answered before the summative questions completed within the tutorial session.
- Keywords for the lecture:

  initial deflections, residual stresses, global imperfections, imperfections for bracing systems, member imperfections, buckling mode, equivalent horizontal force, tolerances.

Notes for lecturers

- Subject: Imperfections of structures.
- Lecture duration: 60 minutes plus 60 minutes tutorial.
- Keywords: initial deflections, residual stresses, global imperfections, imperfections for bracing systems, member imperfections, buckling mode, equivalent horizontal force, tolerances.
- Aspects to be discussed: types of imperfections, necessity of their introduction into analysis.
- Within the lecturing, the introduction of global and member imperfection should be practised and imperfections for bracing system in a roof of an industrial building as well.
- Further reading: relevant documents www.access-steel.com and relevant standards of national standard institutions are strongly recommended.
- Preparation for tutorial exercise: see examples within the lecture.